Examples

1. Prove that $\lim_{n\to\infty} \frac{n+1}{n^2} = 0$.

2. Prove that $\lim_{n\to\infty} \frac{2n}{n^2+3} = 0$.

3. Prove that $\lim_{n\to\infty} \frac{2n}{n^2-3} = 0$.

4. Prove that $\lim_{n\to\infty} \frac{n^2+2n}{n^3-5} = 0$.

Some Properties of Real Nmbers

The following is assigned for homework.

Proposition. (10.4) Let x, $y \in \mathbb{R}$. The $x = y$ if and only if $\forall \varepsilon > 0$ we have $|x - y| \leq \varepsilon$.

Some properties of limits

Theorem If a sequence (a_n) converges, then its limit is unique.

Theorem Every convergent sequence must be bounded.

Theorem Algebraic rules for sequences: Let $\lim_{n\to\infty} s_n = s$ and $\lim\limits_{n\to\infty} t_n = t$.

(a) For $k \in \mathbb{R}$, $\lim\limits_{n\to\infty} ks_n = k \lim\limits_{n\to\infty} s_n = ks$.

(b) $\lim_{n\to\infty} (s_n + t_n) = s + t$.

(c) $\lim_{n\to\infty} (s_n \cdot t_n) = s \cdot t$.

(d) For all n, $s_n \neq 0$ and $s \neq 0$, $\lim_{n\to\infty} \frac{1}{s_n} = \frac{1}{s}$ and $\lim_{n\to\infty} \frac{t_n}{s_n} = \frac{t}{s}$.

Definition

(1) If $\forall M > 0$, $\exists N$ such that $\forall n > N$, $n \in \mathbb{N}$, $s_n > M$, then the sequence diverges to $+\infty$. We write $\lim_{n\to\infty} s_n = +\infty$.

(2) If $\forall M < 0$, $\exists N$ such that $\forall n > N$, $n \in \mathbb{N}$, $s_n < M$, then the sequence diverges to $-\infty$. We write $\lim_{n\to\infty} s_n = -\infty$.

Examples

1. Give a formal proof that $\lim_{n \to \infty} (\sqrt{n} + 7) = +\infty$.

2. Prove that $\lim_{n \to \infty} \frac{n^2 + 4}{n + 2} = +\infty$.

3. Prove that $\lim_{n \to \infty} \frac{n^3}{1 - n} = -\infty$.